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Eccentric Domination in Total Graph of a Graph

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Abstract: Let G be a simple graph with vertex set V(G) and edge set E(G). The total graph T(G) of G is a graph with vertex set $V(G) \cup E(G)$ where two vertices are adjacent if and only if they are adjacent vertices of G or they are adjacent lines of G or one is a vertex of G and another is a line of G incident with it. In this paper we studied the concept of eccentric domination number of total graph T(G), obtained bounds of this parameter and determined its exact value for several classes of graphs.
Keywords: Eccentric domination number, Total graph.

1.Introduction

Let G be a finite simple, undirected graph on p vertices with vertex set V(G) and edge set E(G). For graph theoretic terminology refer to Harary[3], and Kulli[6].

A vertex and an edge are said to cover each other if they are incident. A set of vertices which covers all the edges of a graph G is called a point cover for G, while a set of edges which covers all the vertices is a line cover. The smallest number of vertices in any point cover for G is called its point covering number or simply covering number and is denoted by $\alpha_0(G)$ or α_0. Similarly, α_1 is the smallest number of edges in any line cover of G and is called its line cover number.

The distance d(u, v) between two vertices u and v in G is the minimum length of a path joining them if any; otherwise $d(u, v) = \infty$.

Let G be a connected graph and u be a vertex of G. The eccentricity e(v) of v is the distance to a vertex farthest from v. Thus, $e(v) = \max\{d(u, v): u \in V\}$.The radius r(G) is the minimum eccentricity of the vertices, whereas the diameter diam(G) is the maximum eccentricity. For any connected graph G, $r(G) \leq diam(G) \leq 2r(G)$. The vertex v is a central vertex if e(v) = r(G). The center C(G) is the set of all central vertices. The central sub graph $\langle C(G)\rangle$ of a graph G is the subgraph induced by the center. The vertex v is a peripheral vertex if e(v) = diam(G). The periphery P(G) is the set of all peripheral vertices. For a vertex v, each vertex at a distance e(v) from v is an eccentric vertex. Eccentric set of a vertex v is defined as $E(v) = \{u \in V(G) : d(u, v) = e(v)\}$.

The graph G^+ is obtained from the graph G by attaching a pendant edge to each of the vertices of G. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. Then the union of G_1 and G_2 defined as the graphs $G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$. For three or more graphs G_1, G_2 G_3, …, G_n, the sequential join $G_1 + G_2 + … + G_n$ is the graph $(G_1 + G_2) \cup (G_2 + G_3) \cup … \cup (G_{n-1} + G_n)$.

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The open neighborhood N(v) of a vertex v is the set of all vertices adjacent to v in G. N[v] = N(v) $\cup$ {v} is called the closed neighborhood of v. A graph is self-centered if every vertex is in the center. Thus, in a self-centered graph G all vertex have the same eccentricity, so r(G) = diam(G).

Let G be a simple graph with vertex set V(G) and edge set E(G). The total graph T(G) of G is a graph with vertex set V(G) $\cup$ E(G), where two vertices are adjacent if and only if they are adjacent vertices of G or they are adjacent lines of G or one is a vertex of G and another is a line of G incident with it.

The concept of domination in graphs was introduced by Ore [7]. A set D $\subseteq$ V(G) is said to be a dominating set of G, if every vertex in V(G) $-$ D is adjacent to some vertex in D. D is said to be a minimal dominating set if D $-$ {u} is not a dominating set for any u $\in$ D. The domination number γ(G) of G is the minimum cardinality of a dominating set.

Janakiraman, Bhanumathi and Muthammai [5] introduced the concept of eccentric domination number of a graph. A set D $\subseteq$ V(G) is an eccentric dominating set if D is a dominating set of G and for every v $\in$ V$-$ D, there exists at least one eccentric point of v in D. An eccentric dominating set D is a minimal eccentric dominating set if no proper subset D″$\subseteq$ D is an eccentric dominating set. The eccentric domination number γ_{ed}(G) of a graph G equals the minimum cardinality of an eccentric dominating set. V(G) is an eccentric dominating set for any graph G. Hence, γ_{ed}(G) is an well defined parameter. Obviously, $\gamma(G) \leq \gamma_{ed}(G)$.

Janakiraman [4] proved the following theorems.

Theorem 1.1[4]: Let G be a graph with radius r and diameter d. Then for every v $\in$ V(G), $E_G(v)$ is independent if and only if T(G) is G-eccentricity preserving.

Theorem 1.2[4]: Let G be a self-centered graph with diameter d. Then for each v $\in$ V(G), $E_G(v)$ is independent if and only if T(G) is self-centered with diameter d.

In [5], the following theorems were proved.

Theorem 1.3[5]: $\gamma_{ed}(P_n) = \lceil n / 3 \rceil$, if n = 3k + 1,
$\gamma_{ed}(P_n) = \lceil n / 3 \rceil + 1$, if n = 3k or n = 3k + 1.

Theorem 1.4[5]: (i) $\gamma_{ed}(C_n)$ = n / 2, if n is even.

$$\text{(ii) } \gamma_{ed}(C_n) = \begin{cases} n/3 = \gamma(C_n) \text{ if } n = 3m \text{ is odd} \\ \left\lceil \frac{n}{3} \right\rceil \text{ if } n = 3m+1 \text{ and is odd} \\ \left\lceil \frac{n}{3} \right\rceil + 1 \text{ if } n = 3m+2 \text{ and is odd} \end{cases}$$

Theorem 1.5[5]: $\gamma_{ed}(K_{2n} - F)$ = n.
Theorem 1.6[5]: Let G be a unique eccentric point graph. Then, γ_{ed}(G) = n / 2.

We have already found out the eccentric domination number of total graphs of P_n, C_n, K_n, $K_{1,n}$ and W_n.

Theorem : $\gamma_{ed}(T(P_n)) = \begin{cases} \left\lfloor \frac{2n-4}{5} \right\rfloor + 3, if \ 2n-4 \equiv 4 \ (\text{mod}\,5) \ or \ 2n \equiv 3 (\text{mod}\,5). \\ \left\lfloor \frac{2n-4}{5} \right\rfloor + 2, otherwise. \end{cases}$

Theorem :(i) $\gamma_{ed}(T(C_n)) = n$, n is odd

(ii) $\gamma_{ed}(T(C_n)) = \begin{cases} \frac{n}{2} \ if, n = 4k+2, n \ is \ even \\ \frac{n}{2} + 1 \ if, n \equiv 0 \ (\text{mod}\,4), \ n \ is \ even \end{cases}$

Theorem: $\gamma_{ed}(T(K_{1,n})) = \begin{cases} 2, \ if \ n = 2 \\ 3, \ if \ n \geq 3 \end{cases}$

Theorem: $\gamma_{ed}(T(K_n)) = \begin{cases} 3, if \ n = 3 \\ 4, if \ n = 4 \\ \left\lceil \frac{n}{2} \right\rceil if \ n = 2k+1 is \ odd \\ \frac{n}{2} if \ n = 2k \ is \ even \end{cases}$

Theorem: $\gamma_{ed}(T(W_n)) = \begin{cases} 4, \ if \ n = 3 \\ 5, \ if \ n = 5 \\ \left\lceil \frac{n}{3} \right\rceil + 1, n \ is \ odd \\ \left\lceil \frac{n}{3} \right\rceil + 2, n \ is \ even \end{cases}$

2. Eccentricity properties of T(G)

In this section, eccentricity properties of vertices of T(G) are studied. Radius and diameter of T(G) are also found out. T(G) is disconnected, whenever G has an isolated vertex. Hence, to study the eccentricity of vertices of T(G), assume that G is a graph without isolated vertices.

Theorem 2.1: Let G be a connected graph with radius r distance d then radius of G is r or r +1 and diameter of T(G) is d or d +1.

Proof: Let $u \in C(G)$, $e(u) = r$. Let $v \in V(G)$ such that $d_G(u, v) = r = rad(G)$. In T(G) distance from u to other point vertices is $\leq r$, distance from u to other line vertices is $\leq r + 1$. Let $x \in V(G)$ such that $e(x) =$

d = diam(G). Distance from x to other point vertices is at most d and distance from x to line vertices is at most d + 1. Hence, radius of T(G) is r or r + 1 and diameter of T(G) is d or d + 1.

From Theorem 1.1[4] and Theorem 1.2[4], we get the following theorems.

Theorem 2.2: If G is a self-centered graph with diameter 2 then eccentricity of every point vertex of T(G) is 2 or 3.

Proof: Let $v \in V(G)$ be a point vertex of G and e(v) = 2 in G. Hence distance of any other point vertex from v is two; distance from v to any other line vertex is 3 or less in T(G). Hence the theorem proved.

Theorem 2.3: Let G be a graph with diam(G) = 2. Then T(G) is 2 self-centered if and only if $G \neq K_{1,n}$, and $N_2(u)$ is independent for all $u \in V(G)$.

Proof: T(G) is 2 self-centered implies that $G \neq K_{1,n}$, since $T(K_{1,n})$ is of radius one. T(G) is self-centered with radius 2. This implies that eccentricity of every point vertex and line vertex is 2 in T(G). Therefore, $d(u, v) \leq 2$ in G, for $u, v \in V(G)$. Since T(G) is 2 self-centered distance between any two vertices is ≤ 2. Suppose $u \in V(G)$ and $e \in E(G)$ such that e is not incident with u then d(e, u) = 2 in T(G). This implies that, e is incident with a neighbor of u. Hence $N_2(u)$ is independent.

Conversely, Let diam(G) = 2, $G \neq K_{1,n}$ and $N_2(u)$ is independent. Since diam(G) = 2 and $G \neq K_{1,n}$ distance between any two point vertices is ≤ 2 and for each vertex u in G there exists an edge e in G such that e is not incident with u. Also, $N_2(u)$ is independent. Hence, d(u, e) = 2 in T(G). Hence eccentricity of point vertices is 2 in T(G). If e_1, e_2 are any two adjacent edges then $d(e_1, e_2) = 1$ in T(G). If e_1, e_2 are non adjacent in G, then there exists $e \in G$ such that e is adjacent to both e_1 and e_2 this implies $d(e_1, e_2) = 2$ in T(G). Hence, eccentricity of vertices of T(G) is 2 self-centered.

Remark 2.1: If there exists $u \in V(G)$ such that $N_2(u)$ is not independent in G with more than one vertex then diam(T(G)) = 3.

Remark 2.2: If $G = K_{m,n}$, then T(G) is 2 self-centered.

Theorem 2.4: T(G) is bi-eccentric with diameter two if and only if $G = K_{1,n}$.

Proof: Assume that T(G) is bi-eccentric with diameter 2, then there exists a vertex x in T(G) with eccentricity one in T(G).

Case(i): x = v is a point vertex.

If v is a point vertex in T(G), eccentricity of v in T(G) is one. Therefore v is adjacent to all vertices in G and incident with all edges in G. Therefore, $G = K_{1,n}$.

Case(ii): x = e is a line vertex.

e is a line vertex, $e = xy \in E(G)$. Therfore e is adjacent to x and y and e is not adjacent to any other point vertices. Hence this case is possible only if $G = K_2$.

Conversely, if G = $K_{1,n}$, clearly T(G) is of radius one.

Theorem 2.5: T(G) is complete if and only if G = K_2.

Proof: If G = K_2, T(G) = C_3 which is self-centered with diameter one.

Conversely: Assume that T(G) is self-centered with diameter one. Hence eccentricity of each vertex is one. This implies that G = K_n, $n \geq 2$. But when $n \geq 3$, T(G) is not complete. Hence G = K_2 = P_2.

Theorem 2.6: If G = W_n, $n \geq 5$ then T(G) is bi-eccentric with diameter 3.

Proof: In T(G) eccentricity of u is two, where u is the central vertex of G. Eccentricity of all other vertices are three. Hence, T(G) is bi-eccentric with diameter 3.

Remark 2.3: If G = W_3, then T(G) is 2 self-centered with diameter 2.

Remark 2.4: If G = W_4, then T(G) is 2 self-centered with diameter 2.

Theorem 2.7: If r(G) = 1, diam(G) = 2, then eccentricity of any point vertex in T(G) is 1, 2 or 3.

Proof: Let u be a point vertex. In T(G), d(u, e) $\leq$ 3, where e is any line vertex of T(G). Also d(u, v) = 1 in T(G) if d_G(u, v) = 1 and d(u, v) = 2 in T(G) if d_G(u, v) = 2, where v is any other point vertex of T(G). If d_G(u, v) = 2 and there exists $e_1 \in$ E(G) such that e_1 = xy such that d(u, x) = d(u, y) = 2, then d(u, e_1) = 3 in T(G). Hence, the theorem is proved.

Theorem 2.8: If G is a graph with r(G) = 1 and G $\neq K_{1,n}$, then T(G) is bi-eccentric with diameter 3.

Proof: Let u be a vertex in G. In T(G), distance from u to all other vertices is equal to 2 or 3. Let x $\in$ V(G) be the central vertex of G. In T(G), x is adjacent to all point vertices and line vertices which are incident with x in G and x is at distance 2 to other line vertices. This implies that T(G) is bi-eccentric with diameter 3.

Theorem 2.9: If G is 2 self-centered, then eccentricity of vertices of T(G) is 2 or 3.

Proof: Let diam(G) = 2, and let e(u) = 2 in G. In T(G), d(u, v) = 2, where v is an eccentric vertex of u. Let uwv be a path in G. If there exists e in E(G) which is not in a shortest path from u to v, then in T(G), d(u, e) = 3, otherwise 2.

Corollary 2.9: If G is two self-centered and N_2(u) is totally disconnected for all u $\in$ V(G), then T(G) is 2 self-centered.

Theorem 2.10: T(G) is 3 self-centered if and only if G is any one of the following:
(i) G is 2 self-centered and N_2(u) is not independent for all u $\in$ V(G).
(ii) G is 3 self-centered and N_3(u) is independent for all u $\in$ V(G).
(iii) G is bi-eccentric with diameter 3 such that N_3(u) is independent if e(u) = 3 in G and N_2(v) is not independent if e(v) = 2 in G .

Remark 2.5: If $G = K_n$, $n \geq 3$, then T(G) is 2 self-centered.

Remark 2.6: Let $G = C_n$, then T(G) is self-centered with diameter $\lceil n/2 \rceil$.

Remark 2.7: $G = P_n$, then r(T(G)) = r(G) and diam(T(G)) = diam(G).

Remark 2.8: If G is a tree, then eccentricity of any point vertex u in T(G) is same as the eccentricity of u in G. That is $e_{T(G)}(u) = e_G(u)$, radius of T(G) is r(G) and diamT(G) is diam(G).

Remark 2.9: If $G = F_n = P_n + K_1$, $n \geq 2$, then T(G) is 2 self-centered when n = 2 or 3 and T(G) is bi-eccentric with diameter 3 if $n \geq 4$.

3. Eccentric domination in total graph T(G)

In this section we have studied eccentric domination in total graph T(G) and bounds for $\gamma_{ed}(T(G))$. First, we shall find out the exact value of $\gamma_{ed}(T(G))$ for some particular classes of graphs.

Theorem 3.1: (i) $\gamma(T(K_{m,n})) = m$ if $2 \leq m < n$
(ii) $\gamma_{ed}(T(K_{m,n})) = m$ if $2 \leq m < n$

Proof: Let $V((K_{m,n})) = \{u_1, u_2, u_3, \ldots, u_m, v_1, v_2, v_3, \ldots, v_n\}$ and $E((K_{m,n})) = \{e_{ij} / 1 \leq i \leq m, \quad 1 \leq j \leq n\}$ where $e_{ij} = u_i v_j$ for all $1 \leq i \leq m$, $1 \leq j \leq n$ and let u_{ij} be the added vertices corresponding to the edges e_{ij} of $K_{m,n}$ to obtain $T(K_{m,n})$. Thus $V(T(K_{m,n})) = \{ u_1, u_2, u_3, \ldots, u_m, v_1, v_2, v_3, \ldots, v_n , u_{ij} / 1 \leq i \leq m , 1 \leq j \leq n\}$. $D = \{u_1, u_2, u_3, \ldots, u_m\}$ is a minimum dominating set of $T(K_{m,n})$. Hence, $\gamma(T(K_{m,n})) = m$.

Eccentricity of every point vertex and line vertex of $T(K_{m,n})$ is two. Therefore it is a 2 self-centered graph. Consider $D = \{u_1, u_{22}, u_{33}, u_{44}, \ldots, u_{mn}\}$. D is a minimal eccentric dominating set of $T(K_{m,n})$ and $|D| = m$. Hence, $\gamma_{ed}(T(K_{m,n})) \leq m$. We have $\gamma(G) \leq \gamma_{ed}(G)$. Hence, $\gamma_{ed}(T(K_{m,n})) \geq m$. Therefore, $\gamma_{ed}(T(K_{m,n})) = m$.

Theorem 3.2: (i) $\gamma(T(P_n^+))$ = n , if $n \geq 2$

(ii) $\gamma_{ed}(T(P_n^+))$ = n , if $n \geq 2$

Proof: Let $G = P_n^+$ be a graph obtained from P_n by attaching exactly one pendant edge at each vertex of P_n. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices and $e_{12}, e_{23}, e_{34}, \ldots, e_{n-1,n}$ be the edges in P_n, where $e_{i,i+1} = v_i v_{i+1}$, i = 1, 2, 3, … , n-1. Let u_i be the pendant vertex attached to v_i in P_n^+, i = 1, 2, 3, …, n. Then $v_1, v_2, v_3, \ldots, v_n, u_1, u_2, u_3, \ldots, u_n, e_{11}, e_{22}, e_{33}, \ldots, e_{nn}, e_{12}, e_{23}, e_{34}, \ldots, e_{n-1,n} \in V(T(P_n^+))$. Thus $|V(T(P_n^+))| = 4n - 1$. P_n^+ has n vertices of degree 1, 2 vertices of degree 2 and (n – 2) vertices of degree 3. P_n^+ and $L(P_n^+)$ are subgraphs of $T(P_n^+)$. Let $D = \{v_1, v_2, v_3, \ldots, v_n\}$. D is a γ-set of P_n^+. This D is a point cover for P_n^+ . Hence, $\gamma(T(P_n^+))$ = n.

$D = \{u_1, v_2, v_3, \ldots, v_{n-1}, u_n\}$. u_1 and u_n are two peripheral vertices of T(G). D is an eccentric dominating set of $T(P_n^+)$. Therefore, $\gamma_{ed}(T(P_n^+)) \leq n$. Also $\gamma(T(P_n^+)) = n$. Hence, $\gamma_{ed}(T(P_n^+)) = n$.

Theorem 3.3: (i) $\gamma_{ed}(T(C_n^+)) = 4$, If n = 3

(ii) $\gamma_{ed}(T(C_n^+)) = \left\lceil \frac{4n}{3} \right\rceil$, if $n \geq 4$

Proof: Let G = C_n^+ be a graph obtained from C_n by attaching exactly one pendant edge at each vertex of C_n. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices and $e_{12}, e_{23}, e_{34}, \ldots, e_{n1}$ be the edges in C_n, where $e_{i,i+1} = v_iv_{i+1}$, $1 \leq i \leq n-1$ and $e_{n1} = v_nv_1$. Let u_i be the pendant vertex attached to v_i in C_n^+, i = 1, 2, …, n, where $e_i = u_iv_i$, $1 \leq i \leq n$. Then $v_1, v_2, v_3, \ldots, v_n, u_1, u_2, u_3, \ldots, u_n, e_1, e_2, e_3, \ldots, e_n, e_{12}, e_{23}, e_{34}, \ldots, e_{n1} \in V(T(C_n^+))$. Thus $|V(T(C_n^+))| = 4n$. C_n^+ has n vertices of degree 1, and n vertices of degree 3. C_n^+ and $L(C_n^+)$ are induced subgraphs of $T(C_n^+)$.

Case(i): n = 3

D = $\{u_1, u_3, v_2, v_3\}$ is a minimum eccentric dominating set of $T(C_n^+)$. Hence**,** $\gamma_{ed}(T(C_n^+)) = 4$.

Case(ii): If n = 2k + 2, n is even

The vertex $u_i \in V(T(C_n^+))$ has u_{i+k+1}**,** e_{i+k+1} as eccentric vertices, $v_i \in V(T(C_n^+))$ has u_{i+k+1}, e_{i+k+1} as eccentric vertices and $e_{ii} \in V(T(C_n^+))$ has u_{i+k+1} as eccentric vertex, each vertex has exactly one eccentric point vertex.

Case(iii): n = 2k + 1, n is odd

Each vertex of $T(C_n^+)$ has exactly 5 eccentric points. The vertex $u_i \in V(T(C_n^+))$ has u_{i+k+1}, u_{i+k+2}, e_{i+k+1}, e_{i+k+2}, $e_{i+k+1,\, i+k+2}$ as eccentric points. The vertex $v_i \in V(T(C_n^+))$ has u_{i+k+1}, u_{i+k+2}, e_{i+k+1}, e_{i+k+2}, $e_{i+k+1,\, i+k+2}$ as eccentric points. $e_{ii} \in V(T(C_n^+))$ has u_{i+k+1}, u_{i+k+2} as eccentric points $e_{i,i+1} \in V(T(C_n^+))$ has u_{i+k+2} as eccentric point.

Consider D = $\{u_1, u_2, u_3, \ldots, u_n\} \cup S$, where S is a dominating set of $L(C_n) = C_n$. D is a minimal eccentric dominating set of $T(C_n^+)$. Hence, $\gamma_{ed}(T(C_n^+)) \leq n + \lceil n / 3 \rceil = \lceil 4n / 3 \rceil$. Further, any dominating set of $T(C_n^+)$ must contains at least one of u_i or v_i for all i and a dominating set of L(G). Hence, $\gamma_{ed}(T(C_n^+)) \geq \lceil 4n / 3 \rceil$. Thus, $\gamma_{ed}(T(C_n^+)) = \lceil 4n / 3 \rceil$.

Theorem 3.4: (i) $\gamma(T(K_{2n} - F)) = n$.

(ii) $\gamma_{ed}(T(K_{2n} - F)) = n$.

Proof: Let $K_{2n} - F$ be a complete graph with 2n vertices and $\frac{2n(2n-1)}{2}$ edges. The graph $K_{2n} - F$ is obtained from K_{2n} by deleting n independent edges which form a 1 – factorization or perfect matching. Let $v_1, v_2, v_3, \ldots, v_{2n}$ be the vertices and $e_{ij} = v_iv_j$ (i < j = 1, 2, … ,2n) be the edges in K_{2n}. Let F = $\{e_{1,n+1}, e_{2,n+2}, .., e_{n,2n}\}$. $K_{2n} - F$ has $\left(\frac{4n^2 - 2n}{2}\right) - n = 2n^2 - 2n$ edges. $K_{2n} - F$ is a two self-centered unique eccentric point graph. Hence, $\gamma_{ed}(K_{2n} - F) = n$. The graph $T(K_{2n} - F)$ has $2n + (2n^2 - 2n) = 2n^2$ vertices. D = $\{e_{12}, e_{34}, e_{56} \ldots, e_{2n-1,2n}\}$ is a minimum dominating set of $T(K_{2n} - F)$. Therefore, $\gamma(T(K_{2n} - F))$ = n.

D = $\{e_{12}, e_{34}, e_{56}, \ldots, e_{2n-1,2n}\}$ is an eccentric dominating set of $T(K_{2n} - F)$.

Therefore, $\gamma_{ed}(T(K_{2n} - F)) \leq n$ ……….(1)

We have $\gamma(G) \leq \gamma_{ed}(G)$. Hence, $\gamma_{ed}(T(K_{2n} - F)) \geq n$ ………. (2)

From (1) and (2), $\gamma_{ed}(T(K_{2n} - F)) = n$.

Now, we shall give some bounds for $\gamma_{ed}(T(G))$.

In T(G), a point vertex u has a point vertex v as an eccentric point in T(G) (where d(u, v) = e(v)) or a line vertex e which is incident with an eccentric vertex v of u in G is an eccentric point in T(G). For line vertex e in T(G), another line vertex e_1 or a point vertex v (where $e_1 = vw \in E(G)$) is an eccentric point in T(G).

Theorem 3.5: Let G be a graph without isolated vertices. Set of all point vertices is an eccentric dominating set of T(G), $2 \leq \gamma_{ed}(T(G)) \leq n$. Set of all line vertices is an eccentric dominating set of T(G), $2 \leq \gamma_{ed}(T(G)) \leq m$, where m is the number of edges in G. Thus $\gamma_{ed}(T(G)) \leq \min\{n, m\}$.

Theorem 3.6: Let $n \geq 3$. If G has no isolated vertices with radius 1 and diameter 2, then $\gamma_{ed}(T(G)) \leq \lceil (n + 1) / 2 \rceil$.

Proof: Let u be a central vertex. e(u) = 1 in G. In T(G), u dominates all point vertices and line vertices incident with u in G. Remaining line vertices are dominated by a point cover S of G − u and $|S| \leq \lceil (n - 1) / 2 \rceil$. $S \cup \{u\}$ is an eccentric dominating set of T(G). Therefore, $\gamma_{ed}(T(G)) \leq 1 + \lceil (n - 1) / 2 \rceil = \lceil (n + 1) / 2 \rceil$.

Theorem 3.7: If G is a unicentral tree of radius 2, then $\gamma_{ed}(T(G)) \leq n - \deg_G(u)$, where u is the central vertex of G.

Proof: Let G be a tree of radius 2 with central vertex u. In this case V(G) − N(u) dominates all point vertices and line vertices in T(G). Each vertex in V(T(G)) − (V(G) − N(u)) has eccentric vertices in V(G) − N(u). Therefore, V(G) − N(u) is an eccentric dominating set of T(G). Hence, $\gamma_{ed}(T(G)) \leq n - \deg_G u$.

Theorem3.8: For a bi-central tree with radius 2, $\gamma_{ed}(T(G)) = 4$.

Proof: Let u and v be the central vertices of T. In T(G), N(u) and N(v) are dominating set of T(G). Let x, y be any two peripheral vertices at distance 3 in G. D = {u, v, x, y} form an eccentric dominating set of T(G). Hence, $\gamma_{ed}(T(G)) = 4$.

Theorem 3.9: If G is a spider such that each leg has length 2, then $\gamma_{ed}(T(G)) = \Delta(G) + 2$.

Proof: Let u be the central vertex. $\Delta(G)$ = deg u. |N(u)| vertices form a dominating set of T(G). Adding two peripheral point vertices form an eccentric dominating set of T(G). Hence, $\gamma_{ed}(T(G)) = \Delta(G) + 2$.

Theorem 3.10: If G is a wounded spider, then $\gamma_{ed}(T(G)) = s + 3$, where s is the number of non-wounded legs.

Proof: Let G be a wounded spider. Let u be the central vertex with maximum degree $\Delta(G)$. Let S be the set of support vertices which are adjacent to non-wounded legs in G. In T(G), $S \cup \{u\}$ dominate all point and line vertices. Adding two peripheral point vertices form a minimum eccentric dominating set of T(G). Hence, $\gamma_{ed}(T(G)) = |S| + 3 = s + 3$.

Theorem 3.11: Let G be a tree, then $\gamma(T(G)) \leq \gamma_{ed}(T(G)) \leq \gamma(T(G)) + 2$.

Proof: Let $D \subseteq V(T(G))$ be a γ - set of T(G). Let u, v $\in$ V(G) such that u and v are peripheral vertices of G at distance = diam(G) to each other. Then u or v is an eccentric point of other vertices in G. Again u or v is an eccentric point of line vertices and point vertices in T(G) also. Therefore $S = D \cup \{u, v\}$ is a γ_{ed} - set of T(G). Hence, $\gamma_{ed}(T(G)) \leq \gamma(T(G)) + 2$. Also, we know that $\gamma(G) \leq \gamma_{ed}(G)$ for any graph G. Thus, $\gamma(T(G)) \leq \gamma_{ed}(T(G)) \leq \gamma(T(G)) + 2$.

Theorem 3.12: Let G be a tree with radius > 2. Then $\gamma(T(G)) \leq n - \Delta(G)$ and $\gamma_{ed}(T(G)) \leq n - \Delta(G) + 1$.

Proof: Let G be a tree. Let v $\in$ V(G) such that deg v = $\Delta(G)$. Consider D = V $-$ N(v). In T(G), the vertex v dominates all the point vertices of N(v) and all line vertices incident with v. The point vertices in V $-$ N(v) dominate all other line vertices of T(G). Hence D = V $-$ N(v) is a dominating set for T(G). Thus, $\gamma(T(G)) \leq n - \Delta(G)$.

Case(i): If v is not a support vertex, then clearly D is also an eccentric dominating set of T(G).

Case(ii): Let v be a support vertex. Let w $\in$ N(v) such that w is a pendant vertex, which is a peripheral vertex of G. $D = (V - N(v)) \cup \{w\}$, where w $\in$ N(v) is an eccentric dominating set of T(G). Hence, $\gamma_{ed}(T(G)) \leq n - \Delta(G) + 1$.

Corollary 3.12: Let G be a graph with radius > 2 and u $\in$ V(G) such that N(u) is independent and w $\in$ N(u) is not an eccentric vertex of any other vertex. Then, $\gamma_{ed}(T(G)) \leq n - \deg u + 1$.

Theorem 3.13: Let $G \neq K_{1,n}$ be a graph without isolated vertices. Then a γ_{ed}-set $D \subseteq V(G)$ of G is a γ_{ed}-set of T(G) if and only if D is a point cover of G.

Proof: Let D be γ_{ed}-set of G, which is also a point cover of G. This implies that, D dominates all point and line vertices of T(G). Therefore, D is a dominating set of T(G). Every point vertices not in D has eccentric point vertex in D. Consider e $\in$ E(G). Let e = xy, x, y $\in$ V(G). The eccentric vertex e in T(G) is a point vertex z which is an eccentric vertex of x or y in G. Hence, D is also an eccentric dominating set of T(G).
Conversely, Let $D \subseteq V(G)$ be a γ_{ed}-set of T(G). This implies that, each edge in G is incident with some vertex in D. Therefore, D is a point cover of G.

3.References:

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